

**Statistics**  
**Ph.D. Preliminary Examination**  
**1999**

(Open book, open notes, but No old Exams and solutions)

Name \_\_\_\_\_

1. **(20 points)** Two coins are flipped 100 times. Coin A is an unbiased coin so the probability of flipping a head is 0.5. Coin B is biased and the probability of flipping a head is 0.4.
  - a) What is the probability that at least 30 heads come up in 100 flips? Compute for both coin A and Coin B.
  - b) What is the probability that more heads come up for coin A than coin B in those 100 flips?
  
2. **(20 points)**
  - a) In a school district, the ACT scores follow a normal distribution  $\sim N(22, 4^2)$ . A random sample of 30 students' ACT scores is taken. What is the probability that the sample average will be observed between 20 and 24?
  - b) In another district, a sample of 30 students ACT scores is taken and it is found that the sample average is 21.5, the sample standard deviation is 3.8. Now assume that the individual ACT score in this district,  $X_i$  is normally distributed. What would be the probability that a single  $X_i$  would be observed in the interval (20, 23)?
  
3. **(20 points)** Consider random sampling from a normal population with mean  $\mu$  and variance  $\sigma^2$ .
  - a) Derive a confidence interval of  $\mu$  when  $\sigma^2$  is unknown.
  - b) Derive a confidence interval of  $\sigma^2$  when  $\mu$  is known.
  
4. **(20 points)** The Kent Food Company wishes to test four different package designs for a new breakfast cereal. The features of 4 designs are summarized by the following table:

Package Design	Characteristics
1	3 colors, with cartoons
2	3 colors, without cartoons
3	5 colors, with cartoons
4	5 colors, without cartoons

Ten stores, with approximately equal sales volumes, were selected as experimental units and are randomly assigned one of the package designs. Other relevant condition besides package design, such as price, shelf space and so on, are kept the same for all stores. Sales, in number of cases, were observed for the study period and recorded in the following table:

Package Design	Trials			Total	Mean
	1	2	3		
1	12	18		30	15
2	14	12	13	39	13
3	19	17	21	57	19
4	24	33		54	27
All designs				180	18

Then the following ANOVA table is obtained:

ANOVA table

SOURCE	SS	df	MS
Designs	258	3	86
Error	46	6	7.67

- a) Test the hypothesis that all designs yield equal sale, use  $\alpha = 0.01$ .  
b) Use orthogonal contrast to compare i) 3 color design vs. 5 color designs; ii) Design with cartoon vs. design without cartoon, what are your conclusions?  
c) Conduct a Duncan multiple range test, what is your conclusion?

5. (20 points) Suppose that in a problem of simple linear regression, the ten pairs of observed values of  $x_i$  and  $y_i$  for  $i=1, \dots, 10$  are obtained.

i	1	2	3	4	5	6	7	8	9	10
$x_i$	0.3	1.4	1.0	-0.3	-0.2	1.0	2.0	-1.0	-0.7	0.7
$y_i$	0.4	0.9	0.4	-0.3	0.3	0.8	0.7	-0.4	-0.2	0.7

Linear regression model

$$y = \beta_0 + \beta_1 x + \varepsilon$$

is assumed.

Also, the following computer printout is obtained:

ANOVA table				
Source	DF	Sum of Squares	Mean squares	F-test
Regression	1	1.59	1.59	28.224
Residual	8	0.451	0.056	
Total	9	2.041		

R-Squared: 0.779

Beta Coefficient table			
Parameter	Value	StDev	t-Value
Intercept	0.147	0.083	1.78
Slope	0.435	0.082	5.313

a) Test the hypothesis  $H_0: \beta_1 = 0$ ,

$$H_1: \beta_1 \neq 0$$

- c) Construct a 90% confidence interval for  $\beta_1$   
d) Construct 95% prediction interval for new observation when  $x=1.5$ .