Ph.D. Preliminary Exam - Probability - Fall 2002

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1. Magic Inc buys seventy percent of its sensors from Smart Inc and the rest from All-Star Inc. The lifetime of sensors from Smart Inc. is exponentially distributed with a mean lifetime of 20,000 hours. The lifetime of sensors from All-Star Inc. is exponentially distributed with a mean lifetime of 15,000 hours.
   a) What is the probability that a randomly selected sensor will last more than 20,000 hours? **(5 pts.)**
   b) A sensor failed in less than 15,000 hours. What is the probability the sensor came from All-Star? **(5 pts.)**
   c) A box of five sensors arrived unlabeled. In the end of all of these sensors lasted more than 20,000 hours. What is the probability that the box came from Smart Inc? **(5 pts.)**
   d) If ten sensors from Smart Inc were installed all at once in a complex system. What is the probability that the first sensor will fail in less than 2,400 hours? **(5 pts.)**

2. A sample of twenty weeks of data for demand of two products is listed below. Let X represent the random variable corresponding to product type 1 and Y correspond to product type 2.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
X	2	5	3	4	8	4	9	5	4	7	6	8	3	9	12	5	1	7	3	5
Y	4	8	4	5	5	9	8	6	6	9	8	9	7	4	8	7	9	12	5	6

   Use to the above data to
   a) Estimate $P(Y > X)$ **(5 pts.)**
   b) Estimate $P(Y \geq 6 \mid Y < X)$ **(5 pts.)**
   c) Estimate $E(Y \mid Y > X)$ **(5 pts.)**
   d) Estimate $P(X \geq 7 \cup Y \geq 7)$ **(5 pts.)**

3. Donit is planning a rush order for critical components from a supplier with relatively low quality. The probability that any individual component from this supplier will function as needed is only 0.6. Donit needs four working components. How many components does he need to order so that with 90% certainty he will receive at least four working components? **(10 pts.)**

4. a) Assume a Poisson distribution with parameter λ. Determine the probability that the random variable is at least double the average for $\lambda=1$, $\lambda=2$, $\lambda = 4$, $\lambda=6$, $\lambda=9$. For example when $\lambda=1$, find $P(X \geq 2)$ and repeat for other λ values. Describe the pattern that you see and provide a justification based on the standard deviation. **(5 pts.)**

b) Assume a continuous exponential distribution with parameter α. The mean is equal to α (not 1/ α). Determine the probability that the random variable is at least double the average for α =1, α=2, α = 4, α =6, α =9. For example when α =1, find $P(X \geq 2)$. Describe the pattern that you see and provide a justification. **(5 pts.)**

5. A plane can hold 150 people. The probability distribution for the weight of the baggage an individual brings with him is normally distributed with a mean of 50 pounds and a standard deviation of 20 pounds. What is the probability that the combined weight of the baggage will exceed 8000 pounds? **(5 pts.)**

6. a) The number of fatal accidents in a three-state region on a holiday weekend is Poisson distributed with a mean of 75. The state police are working hard this year to reduce the fatal accident rate. At the end of the weekend, they found there were only 70 fatalities. Do these data provide a strong indication that the police activities had an affect in reducing fatal accidents? **(5 pts.)**
   b) Assume there are six holiday weekends during the course of the year and, in fact, police activities are having no significant impact. What is the probability that the first time the number of fatalities drops to 70 or less is on the third holiday weekend of the year? **(5 pts)**
   c) Assume there are six holiday weekends during the course of the year. Which of the following two data points would be the stronger evidence of police effect and why? **(10 pts.)**
   1) On each of the six holiday weekends the number of fatalities was 70 or less?
   2) The total number of fatalities during the combined six holiday weekends was less than 400

   Remember the normal distribution can be used to approximate the Poisson distribution as well as the binomial.

7. Assume there is a warranty cost incurred if a machine fails before time T.
   a) How large would T have to be if you want 98% of the machines **<u>not</u>** to incur a warranty cost? The time to failure is exponentially distributed with a mean of 180 days? **(5 pts.)**
   b) Now assume the warranty extends for 100 days. When a machine fails, the average cost of repair is $75. (Assume machines almost never fail more than once during the warranty period.) On average how much does the company incur in warranty costs per machine? **(5 pts.)**
   c) There is a one-to-one correspondence between Poisson counting process and the time between occurrences following the exponential distribution. With an exponential failure distribution with a mean of 180 days, what is the probability the machine will fail exactly twice during the first 100 days? **(5 pts.)**