# Ph.D. Preliminary Exam - Probability - Fall 2001

Kenneth Chelst, Ph.D.
Industrial & Manufacturing Engineering, Wayne State University

1. You are given the following joint probability density function (p.d.f.)

   $f(x,y) = x - y \qquad 0 < y < 1$ and $1 < x < 2$

   a. Demonstrate or prove that the random variables X and Y are NOT independent. (5 pts)

   Find each of the following for the above p.d.f.

   b. $P(X < 1.5)$ (5 pts)

   c. $P(X<1.5 \mid Y \leq 0.9)$ (5 pts)

   d. $P(X<1.5 \mid Y = 0.9)$ (5 pts)

   e. Find F(X) the cumulative distribution function of X. (5 pts)

2. The rate of defects in a particular part is relatively low, an average of one defect per one thousand parts. What is the probability that a shipment of twenty-five thousand parts contains 20 or fewer defective parts? (5 pts)

3. A new high-tech testing device is designed to rapidly identify whether or not a computer chip is defective. The operators who use the testing device sometimes make errors. If a chip is <u>not defective</u>, the test performed by the operators will indicate <u>not defective 98% of the time</u>. If a chip is <u>defective</u>, the test performed by the operators will indicate <u>defective 99% of the time</u>. Historically, one out of every five thousand chips was actually defective.

   a. The test results on a specific chip suggest the chip is defective. What is the probability that the chip is in fact defective? (5 pts)

   b. Each time the testing device is used the chances of an error is independent of the previous test even when performed on the same computer chip. A decision was made to retest the chip that the test had indicated was defective in part a) above. The test results come back and again indicate the part is defective. After these two test results both indicate the chip is defective, what is the probability that the chip is in fact defective? (5 pts)

4. Data on the time it takes to extract a small amount of an enzyme from a blood sample are presented in the table below. (The data are in hours and represent 200 blood samples.)

| Time (hours) | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| Number | 15 | 25 | 40 | 50 | 40 | 30 |

   a. On average how long does it take to extract the enzyme? (5 pts)

   The cost (in dollars) for extracting the enzyme is a function of the time, T.

   $\text{Cost} = 10\,T^2 + 50$

   b. What is E(Cost), the expected value of the Cost? (5 pts)
   c. What is the probability that the cost of extraction will be less than $241? (5 pts)
   d. A lab technician has been working on a specific sample for 3 hours already and he has not yet succeeded in extracting the enzyme. What is the probability that he will complete the task in a total of 5 hours or less? (5 pts)

5. The time to process a medical claim is uniformly distributed between 4 and 5 hours and the time is independent of which worker is processing the claim.
    a. Ten workers are each given a claim to process. What is the probability that all of them finish processing their claims in under 4.75 hours? (5 pts)
    b. Ten workers are each given a claim to process. What is the probability that two or fewer workers complete processing their claims in under 4.20 hours? (5 pts)
    c. An individual worker is given a batch of 100 claims to process. What is the probability he will complete processing of the claims in 425 hours or less? (5 pts)

6. The number of spots on a photographic sheet is Poisson distributed with a mean of 1 spot per 100 $cm^2$.
    a. What is the probability that there is exactly one spot on a sheet that is 50 $cm^2$? (5 pts)
    b. What is the probability that in a large sheet that is 10,000 $cm^2$, that there are 140 or fewer spots? (5 pts)

7. The lifetime of a component is exponentially distributed with a mean of 5,000 hours of operation.
    a. The manufacturer is considering warranting the part and offering to replace it free of charge if it fails before "X" hours. The manufacturer wants to know what he should set "X" to be so that 90% of the parts will NOT have to be replaced free of charge? (5 pts)
    b. A specific component continues to function after 6,000 hours. What is the probability that the component continues to function even after 10,000 hours of operation? (5 pts)
    c. A purchaser of the component is suspicious of the claim of an average operating lifetime of 5000 hours because so many components he buys do not seem to last that long. What is the probability that the first time one of the components he buys lasts longer than 5000 hours is the third component he bought? (5 pts)