

Ph.D. Preliminary Exam - Probability - Fall 2000

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1-4. The local Luxury Car Rental Company typically purchases a small fleet of three vehicles each week. It inspects and records the number of cars with mechanical and surface defects in each fleet. Let X represent the number of cars with mechanical defects and Y the cars with Surface defects. The probability distribution for this pair of random variables is given below.

		Cars with Surface Defects				
		Y	0	1	2	3
Cars with Mechanical Defects	X					
	0		.10	.15	.09	.08
	1		.08	.08	.06	.07
	2		.06	.04	.03	.04
3		.06	.02	.01	.03	

1. Find the probability that the number of cars with mechanical defects will be less than or equal to the number of cars with surface defects.
2. Find the $P(X + Y \leq 2)$
3. Construct the conditional probability distribution for X, in a week in which it was found that there were two cars with surface defects.
4. What is the conditional expected value of X, in a week in which it was found that there were two cars with surface defects.
5. The probability distribution of the time to failure of a machine is exponentially distributed with a mean of one thousand hours. Ten machines are installed into a plant. What is the probability that at least eight of them are operating after five hundred hours?
6. The probability distribution of the time to failure of a machine is exponentially distributed with a mean of one thousand hours. Ten machines were installed into a plant. Two thousand hours have already passed and nine of the machines have failed. What is the probability that the last machine fails within the NEXT five hundred hours?
7. The probability of successful development of a new drug is approximately one in ten. You have already researched nine different new drugs this year and none were successful. What is the probability that if you research a total 12 drugs this year that at least one will be successful?
8. The probability distribution of the time to failure of a machine is exponentially distributed with a mean of one thousand hours. You receive a report that the machine failed in less than five hundred hours but the exact time of failure was not recorded. Given this information, what is the probability that the machine actually failed in less than two hundred hours?
9. The number of emergency calls in a small city averages two calls per hour. What is the probability that during a three hour period there will be four or fewer calls?

10. The number of emergency calls in a small city averages two calls per hour. What is the probability that in every single hour during a four-hour period there are at least two calls in each and every hour?
11. A precision tool should be as close to 3 cm in length as possible. Parts that are within .01 cm are, however, acceptable. The process for making this tool has variability but the mean is 3 cm. The goal is for 98% of the parts to meet this acceptable specification standard. The company is working to reduce process variability. How low must the standard deviation be in order for the process to meet the 98% goal?
12. Daily demand for a component is normally distributed with a mean of 50 and a standard deviation of 10 parts. How many parts must be in inventory at the beginning of the day in order to meet that day's demand with 0.90 probability?
13. A company has a stockpile of 100 short-lived components. The lifetime of each component is exponentially distributed with a mean of only two hours. When one component fails, it is replaced automatically and instantly from the stockpile. On average this supply of 100 components should last two hundred hours. What is the probability that in fact the supply will last less than 190 hours?
- 14-16. Both the normal distribution with the half-unit correction and the Poisson distribution can be used to approximate the binomial distribution. The next few questions explore differences between these approximations.

14. Assume you have a binomial distribution for a random variable X in which $n=25$ and $p=.40$. Complete the following table.

	Binomial	Poisson Approx.	Normal Approx.
$P(X=8)$			
$P(X \leq 8)$			

15. Assume you have a binomial distribution for a random variable X in which $n=10$ and $p=.10$. Complete the following table.

	Binomial	Poisson Approx.	Normal Approx.
$P(X=0)$			
$P(X \leq 1)$			

16. Please explain which of the approximations to the Binomial seems to be better under what circumstances. You can refer to rules of thumb but I am also looking for an understanding of the motivation behind the rules of thumb for when to use a particular approximation.