

WAYNE STATE UNIVERSITY
Department of Industrial and Manufacturing Engineering

PhD Preliminary Examination – 03 October 2002

Candidate Name: _____

Candidate Number: _____

Answer ALL Questions

Question 1 – 20 Marks

Question 2 – 20 Marks

Question 3 – 15 Marks

Question 4 – 25 Marks

Question 5 – 20 Marks

Question 1 – Large Scale Optimization

Engineers are designing the locations for modules $i=1, \dots, m$ from among the $j=1, \dots, n$ available sites on a computer board. They already know:

$$a_{i,i'} \triangleq \begin{cases} 1 & \text{if a wire is required from} \\ & \text{module } i \text{ to module } i' \\ 0 & \text{otherwise} \end{cases}$$
$$d_{j,j'} \triangleq \text{distance between sites } j \text{ and } j'$$

Using this information, they wish to choose a combination of locations that will minimize the total wire length required.

- a. Explain why appropriate decision variables for a model of this problem are $(i=1, \dots, m; j=1, \dots, n)$

$$x_{i,j} \triangleq \begin{cases} 1 & \text{if } i \text{ goes to site } j \\ 0 & \text{otherwise} \end{cases}$$

- b. Explain why the length of any wire required between modules i and i' can be expressed as:

$$\sum_{j=1}^n \sum_{j'=1}^n d_{j,j'} x_{i,j} x_{i',j'}$$

- c. Use the expression in part (b) to formulate an objective function minimizing total wire length
- d. Formulate a system of m constraints assuring that each module is assigned a location.
- e. Formulate a system of n constraints assuring that each location gets at most one module.
- f. Complete your model with an appropriate system of variable-type constraints.
- g. Is your model best classified as an LP, an NLP, an ILP, or a MILP, and is it single or multi-objective? Explain.

Question 2 – LP Case Analysis

Harry's is a chain of 47 self service gas stations served by a small refinery and mixing plant. Each day's product requirements are met by blending feedstocks on hand at midnight. The volumes vary daily, depending on the previous day's refinery output and on bulk receipts.

The entire operation is run by the owner, Harry Paul. Although dozens of chemicals and byproducts are generated by the refinery, Harry's major concern is the retail distribution of gasoline products.

On a particular Tuesday there are significant volumes of leaded and unleaded regular gasolines at the stations. Only the two hybrid petroleum products – gasohol and petrolmeth – will be shipped that day. Both products are blended from 90-octane unleaded gasoline. Ethyl alcohol, the only additive to gasohol, cannot exceed 10% of the final product's volume. Petrolmeth may contain both ethyl and methyl alcohols, but these combined ingredients must not exceed 30% of the final product's volume. The octane ratings are 120 for ethyl alcohol and 110 for methyl alcohol. Final product octane ratings must equal the average octane ratings for the ingredients by volume. Gasohol must have an octane rating of at least 91, and petrolmeth must have a rating of at least 93.

There are 20,000 gallons of gasoline presently available for blending, at a cost of \$1.00 per gallon. Up to 5000 gallons of methyl alcohol can be acquired for \$.50 per gallon, 3000 gallons of ethyl alcohol are available at \$1.50 per gallon. The demands are at least 10,000 gallons for gasohol and 5000 gallons for petrolmeth.

Until now Harry has determined product blends by trial and error. A new staff analyst says she can save a considerable amount of money by using LP to establish a minimum cost blending formulation. Harry is a bit skeptical, but he offers her the challenge to do better than his method.

Questions

1. Define an appropriate set of decision variables
2. Formulate a symbolic model for this problem
3. Explain how Harry maybe able to use the optimal solution to support blending decisions.

Question 3 – Decision Analysis

A large corporation has found itself with \$2,000,000 in excess funds and is considering one of three options for the use of this money for the coming year:

- 1) purchase preferred certificates of deposit (CD's);
- 2) accept a contract which has already been offered to construct a new pollution control device;
- 3) invest the entire sum in stocks, bonds, or real estate.

If option (1) is selected, then the entire \$2,000,000 will be used and the return is 14%. If the firm decides to build the pollution control device, they will make \$500,000 if they can finish the job in one year. If not, then they will lose the entire \$2,000,000. The R&D department estimates an 85% chance of finishing within the year. If the firm selects option (3), then a subsequent decision must be made. If the \$2,000,000 is invested in stocks, then a yield of 20% will occur if the market goes up. A yield of 2% will occur if the market stays stable and a loss of 8% will occur if the market goes down. For the coming year, probabilities of 0.2, 0.6, and 0.2 have been estimated (respectively) for the market conditions. If the \$2,000,000 is invested in bonds, then a yield of 12% is obtained. If the firm puts the \$2,000,000 into real estate, then they must decide on site #1 or site #2. Site #1 will yield a profit of \$440,000 only if it is rezoned for industrial use. The firm has estimated that there is an 80% chance that this rezoning will occur. If not, then the firm will resell the site and incur a loss of \$100,000. If site #2 is selected, then a profit of \$800,000 will be made if a proposed shopping mall is constructed on the adjacent acreage. However, there is only a 25% chance this will occur. If the shopping mall is not built, then the land is relatively worthless and the firm will lose \$500,000. Finally, the firm could invest \$1,000,000 in site #1 and \$1,000,000 in site #2. Each investment would result in half the expected profit of the corresponding \$2,000,000 investment.

- a. Construct the decision tree for this problem
- b. Find the probabilities for the branches emanating from the chance nodes.
- c. Analyze the decision tree to identify the optimal strategy and what is the final expected total payoff.
- d. Now suppose an individual appeared and claimed that he could predict with certainty whether or not the firm would be able to complete the pollution control device within one year's time. What is the most the firm would be willing to pay this individual?

Question 4 – Sensitivity Analysis

Part a.

The Khan House of Furniture makes two kinds of tables – end tables (x_1) and coffee tables (x_2). The manufacturer is restricted by material and labor constraints, as shown in the following LP formulation.

Maximize $Z = 200x_1 + 300x_2$ (profit, \$)

Subject to

$$2x_1 + 5x_2 \leq 180 \text{ (labor, hr)}$$

$$3x_1 + 3x_2 \leq 135 \text{ (wood, bd ft)}$$

$$x_1, x_2 \geq 0$$

The final optimal simplex for the problem is as follows.

			200	300	0	0
c_j	Basic Variables	Quantity	x_1	x_2	s_1	s_2
300	x_2	30	0	1	1/3	-2/9
200	x_1	15	1	0	-1/3	5/9
	z_j	12,000	200	300	100/3	400/9
	$c_j - z_j$		0	0	-100/3	-400/9

- Formulate the dual for this problem.
- Define the dual variables and indicate their values.
- What profit for coffee tables will result in no end tables being produced, and what will the new optimal solution values be?
- What will be the effect on the optimal solution if the available wood is increased from 135 to 165 board feet?
- Determine the optimal ranges for c_1 and c_2 .
- Determine the feasible ranges for q_1 (labor hours) and q_2 (board feet of wood).
- What is the maximum price the Khan House Furniture Company would be willing to pay for additional wood, and how many board feet of wood could be purchased at that price?
- If the furniture company wanted to secure additional units of only one of the resources, labor or wood, which should it be?

Part b.

Consider the following LP and computer solution:

Let X_i = the number of level i ($i = 1, 2, 3$) staff personnel to hire for a new facility

MIN the hourly cost:

$$\begin{aligned}
 & \text{MIN} && 12X_1 + 18X_2 + 24X_3 \\
 & \text{s.t.} && 2X_1 - X_2 \geq 0 && \text{[Balance]} \\
 & && X_1 + X_2 + X_3 \geq 1000 && \text{[State reg.]} \\
 & && X_1 \geq 400 && \text{[Type 1 need]} \\
 & && X_2 \geq 400 && \text{[Type 2 need]} \\
 & && X_3 \geq 100 && \text{[Type 3 need]} \\
 & && X_2 \leq 500 && \text{[Type 2 avail.]} \\
 & && X_1, X_2, X_3 \geq 0
 \end{aligned}$$

	X1	X2	X3				
Objective	12	18	24				
Decision	500	400	100	OV			
				15600			
Constraints	Resource Usage			LHS	RHS	SLACK	
Balance	2	-1		600	≥	0	600
State Reg.	1	1	1	1000	≥	1000	0
Type 1 Need	1			500	≥	400	100
Type 2 Need		1		400	≥	400	0
Type 3 Need			1	100	≥	100	0
Type 2 Avail		1		400	≤	500	100

Changing Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$3	Decision X1	500	0	12	6	12
\$C\$3	Decision X2	400	0	18	1E+30	6
\$D\$3	Decision X3	100	0	24	1E+30	12

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$E\$8	Balance LHS	600	0	0	600	1E+30
\$E\$9	State Reg. LHS	1000	12	1000	1E+30	100
\$E\$10	Type 1 Need LHS	500	0	400	100	1E+30
\$E\$11	Type 2 Need LHS	400	6	400	100	400
\$E\$12	Type 3 Need LHS	100	12	100	100	100
\$E\$13	Type 2 Avail LHS	400	0	500	1E+30	100

- a. Identify the active constraints
- b. What is the minimum hourly costs
- c. Will there be a surplus over and above the basic need of each level of staff personal? If so, which one(s), and how many excess personnel will there be of this type(s)?
- d. Suppose that the manager of the firm were able to reduce the cost for level 2 personnel from \$18/hour to \$12/hour. How many personnel of each type will be hired?
- e. What would happen to the minimum cost if the state regulation were to change from 1000 to 999?

Question 5 – Queuing Models



Peter McGregor is the operations manager for Caledonia Bank. He is establishing teller schedules for the Seventh Street Branch. The following data apply to the arrivals of customers at various times of the week.

Period	Daily Average Number of Arrivals
(1) Monday–Friday 10 A.M.–12 P.M.	155
(2) Monday–Friday 12–1 P.M.	242
(3) Monday–Friday 1–3 P.M.	290
(4) Friday 3–6 P.M.	554

The bank opens at 10 A.M. and closes at 3 P.M., except for Friday, when it closes at 6 P.M. Past study shows that arrivals over each period constitute a Poisson process. The mean time to complete customer transactions is 2 minutes, and individual service times are approximated by an exponential distribution.

Tellers all work part-time and cost \$10 per bank hour. Past experience shows that a significant drop-off in clientele soon follows when customers are forced to experience lengthy waits.

McGregor also needs to decide whether or not to install an automated teller machine (ATM). The equipment supplier claims that other banks experience a 30% diversion of regular business away from human tellers, plus a further 10% expansion in the previous level of overall client transactions (with all of the increase going to the ATM).

ATM business takes place on a 24-hour basis, although traffic is negligible between 11 P.M. and 6 A.M. During the busier hours, times between customer arrivals are assumed to be represented by a single exponential distribution.

The mean service time at an ATM is .5 minute, and again the exponential distribution serves as an adequate approximation. McGregor believes that once an ATM is installed, the human tellers would be

left with a greater proportion of the more involved and lengthy transactions, raising the mean service time to 2.5 minutes.

QUESTIONS

- Assume that Caledonia Bank uses human tellers only.
 - For each time period, determine the minimum number of tellers needed on station to service the customer stream.
 - Assume that the numbers of tellers in (a) are used. For each time period, determine the mean customer waiting time.
 - For each time period, determine the mean customer waiting time when the number of tellers is one more than found in (a).
- Past experience shows that the drop-off in clientele due to waiting translates into an expected net present value in lost future profits of \$.10 per minute. For each time period, determine the average hourly queuing system costs, assuming that the bank uses
 - the minimum number of human tellers necessary to service the arriving customers and
 - one teller more than is found in (a) of Question 1.
- Suppose that the ATM is installed and that customers themselves decide whether to use human tellers or to use the ATM, and that two queues form independently for each. Finally, assume that there is a 10% traffic increase generated by the ATM within each open time period, and that all of it is for the ATM. Determine, in each case, the mean customer arrival rate
 - at the human teller windows and
 - at the ATM. Then, find (c) the minimum number of human tellers required to be on station during each time period.