

**Ph.D. Preliminary Exam – Probability – Feb 2006**

Exam – OPEN BOOK Author – Kenneth Chelst

1. (15 points) Two troublesome machines are being studied as part of a factory productivity improvement plan. Assume both machines have exponentially distributed times to failure and exponentially distributed times to repair. Mean Time to Failure (MTTF) and Mean Time to Repair (MTTR).

Machine #1 has MTTF = 2 hrs and MTTR = 1 hr.

Machine #2 has MTTF = 3.5 hrs and MTTR of 2.5 hrs.

- Assume that both machines are functioning properly. Calculate the probability that neither machine would fail in the next 8 hr period.
- What is the probability that the first machine to fail is machine #1?
- Calculate the expected availability for each machine. In the long run, which machine would have better uptime statistics?

2. (28 points) The probability density function for the time to complete a task is given as follows

$$f(t) = 4t^3/80 \quad 1 < t < 3$$

- What is the expected value of the time to complete a task?
- What is the standard deviation of the time to complete a task?
- The cost in dollars of completing a task is given by the function  $c = 5t^{0.5}$ . What is the average cost of completing a task?
- What is the probability that the cost of completing the task will be less than \$8.90?
- You have been working on the task for 2.5 hours. What is the probability that the task will be completed within the next fifteen minutes?
- Six different workers have been assigned tasks with the same pdf for completion times. What is the probability that the first one to complete the task will do so in less than 1.5 hours?
- Six different workers have been assigned tasks with the same pdf for completion times. What is the probability that exactly four of the six will complete the task in less than 1.5 hours?

3. (15 points) One in a hundred people have a genetic defect X. If the genetic defect is present, a new test will record a positive result 97% of the time. This means there is a 3% False Negative rate. If the genetic defect is NOT present, the test will still yield a positive result 5% of the time. This means there is a False Positive rate of 5%.

- If the tests come back positive, what is the probability the individual has the genetic defect?
- The goal is to make the test so reliable, that if it produces a positive result that likelihood the individual has the genetic defect is at least 0.9. How low would the false positive rate have to be so that they could achieve this goal? Remember it is now 5%.
- Could they achieve the test reliability goal mentioned in part b) by reducing the False Negative rate without reducing the False Positive rate? Explain your answer.

4. (11 points) The length of a bar of steel is normally distributed with a mean 1 meter and a standard deviation of .01 meters.
- What is the probability that 500 bars that are laid end-to-end will be longer than 500.1 meters?
  - Management is working to reduce the variability of the length of a bar. What would the standard deviation have to be such that each randomly selected bar has a 99.9% chance of being between .995 meters and 1.005 meters?
5. (15 points) The number of major plane crashes in the US is Poisson distributed with a mean of one per year.
- What is the probability of there being no major plane crashes for three years in a row?
  - What is the probability of there being at least one major plane crash in each of three years in a row?
  - What is the probability that the time between crashes will be 2.5 years or longer?
6. (16 points) The times (in weeks) to complete each of a sequence of two tasks are random variables X and Y. The times have been approximated as discrete random variables.

		Y			
		1	2	3	4
X	1	.05	.05	0	0
	2	.05	.15	.05	.10
	3	.05	.1	.15	.05
	4	0	.05	.1	.05

- On average how long will it take to complete both tasks?
- What is the probability that it will take longer to complete task X than to complete task Y?
- If task Y takes 2 weeks to complete, on average how long will it take to complete task X?
- Prove that X and Y are NOT independent random variables.