

Preliminary Exam – Probability February 2004
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1. The total number of defects produced each day for an 80-day period is given below.

(24 points)

	Defects					
Number	0	1	2	3	4	5
Days	15	20	22	10	8	5

- a) What is the average number of defects?
- b) What is the variance in the number of defects?
- c) The cost of repair is given by the formula $C = 25 + 20D^{0.5}$ when D is greater than 0. It is zero if there are no defects. What is $E(C)$?
- d) What is the probability that the total cost of repair for a day is \$65 or more?
- e) It was a bad day and they stopped counting after they found the third defect. What is the probability that in fact there were five defects that day?
- f) What is the probability that there were the same number of defects on two consecutive days?

2. You are given the following probability density function for the total demand for gas, X, in a day. It is measured in thousands of gallons. **(12 points)**

$$f(x) = (3x^2)/117 \quad 2 < x < 5$$

- a) What is the probability that the demand on any given day will be less than four thousand gallons?
- b) Determine the cumulative distribution function of X, if X is known to be less than 4.
Find $F(x | x < 4)$.
- c) Describe step by step how you would determine the probability that over a 30 day period the total demand will be less than 120 thousand gallons. Do no calculations.

3. You are provided with the following joint probability density function **(24 points)**

$$f(x,y) = (x + y)/16 \quad 1 < x < 3 \text{ and } 1 < y < 3$$

- a) Find $E(X)$
- b) Find $P(X < 1.7)$
- c) Find $P(X < 1.7 | Y = 1.3)$
- d) Set up the integrals to find $P(X > Y + .2)$ Do not solve.
- e) Determine $F(X)$ the cdf of X.
- f) Determine $E(X - Y)$. Justify the answer you found on an intuitive basis.

4. The time to complete an individual task is exponentially distributed with a mean of 2 hours. The time to complete a task is independent of prior experiences and is always exponentially distributed.

(24 points)

- a) What is the probability that the first time the worker is able to complete the task in less than 1 hour occurs on the fourth repetition?
- b) What is the probability that on exactly two attempts in his first five, the task was completed each time in less 2 hours and the other three times it took longer than 2 hours?
- c) What is the probability that a COMBINED TOTAL of 100 independent repetitions of this task can be completed in less than 205 hours?
- d) How many hours, H , would you have to set aside to be sure with 0.95 probability that a total of 100 repetitions would be completed in under H hours?
- e) Two hundred hours have passed and you have not completed the 100 repetitions. What is the probability that you will complete the 100 repetitions within the next 10 hours?
- f) You have completed 99 repetitions by the end of 199 hours and have been working on the 100th for over an hour. What is the probability that you will finish this final task in **less than one more hour**?

5. The number of emergency calls in a small city is Poisson distributed with a mean of 2 calls per hour.

(16 points)

- a) What is the probability that the next call arrives within twenty minutes?
- b) What is the probability that there is at least one call between 1:30 PM and 1:45PM?
- c) What is the probability that during a five hour period, that during at least one of the hours there will be no calls? (For this problem consider each of the five hours as a separate hour.)
- d) Over the course of an entire 168 hour week, what is the probability that the city will receive more than 350 calls?