

Probability: Preliminary Exam – May 2003

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1. Poisson process and exponential distribution **(28 points – 4 points per question)**
- a) Explain in words the relationship between the Poisson process and the exponential distribution
 - b) Explain in words the meaning of the memoryless property of the exponential distribution.
 - c) A 911 dispatcher in Smallville handles both police calls and emergency medical calls. Throughout the day, police calls arrive at a rate of two per hour and emergency medical calls arrive at a rate of one per hour. The number of calls follows the Poisson distribution. What is the probability that in a three-hour period the dispatcher will handle three or fewer calls?
 - d) What is the probability that there will be two or fewer calls in each hour between 1 PM and 4PM?
 - e) A dispatcher has just arrived on duty. What is the probability that he will receive a call within the first five minutes of arriving on duty?
 - f) What is the probability that during a 48-hour period, there will be a total of more than 150 calls?
 - g) You were told that between 1PM and 2 PM no more than one emergency medical call was received. Given that fact, what is the probability that there were exactly the same number of police calls as emergency medical calls during this hour?

2. The probability density function for the time in hours to complete a task has the following functional form:**(28 points: 4 points per question)**

$$f(t) = 3t^2/117 \quad 2 < t < 5 \quad \text{equation (1)}$$

- a) Find the cumulative distribution function.
- b) The cost in dollars, c , of completing this task is a function $c = 50 + 30t^{0.5}$
What is the expected value of the cost of completing this task?
- c) Refer back to part b. What is the probability that the cost of completing this task will be less than \$110?
- d) You have been working on the task for 3 hours and it is still not completed. On average how much longer will it take to complete the task?
- e) Assume the pdf as stated above in equation (1). However, if at the end of four hours worker A is unable to complete the task, he turns it over to worker B to finish the task. What is the probability that worker A will be unable to complete the task?
- f) If worker A is handed five tasks to do, what is the probability that he will be able to complete four out of the five by himself?
- g) What is the average time that worker A will work on a randomly selected task?

3. A company receives batches of components from two different suppliers. Supplier A provides 30% of the batches and supplier B the other 70%. Supplier A has a defect rate of 20% and supplier B has a defect rate of 10%. **(15 points: 5 points each)**

- A batch of 20 components has just been received from one of these suppliers. On average how many defective components will there be in the batch?
- What is the probability that there will be two or fewer defects in this batch of 20 components?
- The company just received a very large batch of 2,000 components from one of these two suppliers. What is the probability that the batch has fewer than 300 defective components?

4. A bar is to be inserted into a hole. The radius of the hole is normally distributed with a mean of 3.250 cm and a standard deviation of 0.012cm. The diameter of the bar is normally distributed with a mean of 3.240 cm and a standard deviation of 0.005cm. **(15 points: 5 points each)**

- What is the probability that a randomly selected bar will fit inside a randomly selected hole?
- Assume you can invest money to reduce the variation in the cutting of the hole. What would the standard deviation have to be reduced to in order for 90% of the randomly selected bars to fit in a randomly selected hole?
- Now assume that in order for the bar to function properly when inserted in the hole, it must not only fit inside the hole but the clearance cannot be greater than 0.015 cm. (Use the original standard deviations.) What is the probability that a randomly selected bar will function properly inside a randomly selected hole?

5. The joint distribution of the number of cars, C, and trucks, T, that pass through a busy intersection during a one-minute interval is given in the table below. **(14 points: 3.5 points each)**

| | | Number of Cars | | | | | |
|------------------|---|----------------|-----|-----|-----|-----|-----|
| | | C | 0 | 1 | 2 | 3 | 4 |
| Number of Trucks | T | 0 | 0 | .04 | .12 | .09 | .05 |
| | 1 | 0 | .02 | .05 | .05 | .03 | |
| | 2 | .06 | .07 | .10 | .07 | .05 | |
| | 3 | .05 | .10 | .05 | 0 | 0 | |

- What is the probability that the combined total of cars and trucks that pass through the intersection in one minute is less than or equal to 2?
- You noticed that two cars passed through the intersection. What is the probability that two trucks also passed through the intersection?
- What is the combined average number of cars and trucks that pass through the intersection in a one-minute interval?
- What is the probability that an equal number of cars and trucks passed through the intersection in a one-minute interval?