# WAYNE STATE UNIVERSITY

## Department of Industrial and Manufacturing Engineering

PhD Preliminary Examination- February 2006

Candidate Name: ________________________

**Answer ALL Questions**
Question 1- 20 Marks
Question 2- 15 Marks
Question 3- 20 Marks
Question 4- 25 Marks
Question 5- 20 Marks

**1- LP(Linear Programming) Problem**

Nutworks Inc. produces both Narcs and Widgets in plants A and B. Production rates are 2 Narcs per hour and 5 Widgets per hour at plant A and 3 Narcs per hour and 5 Widgets per hour at plant B. The capacity of plant A is 60 hr regular time plus 30 hr overtime and the capacity of plant B is 80 hr regular time and 40 hr overtime in each period. Production cost is $100 per hour regular time and $150 per hour overtime. All production goes into inventory or is shipped to customers. The inventory cost is negligible. Nutworks has received the following orders from customers P and Q for the next two time periods.

| Customer | Product | Period 1 | Period 2 |
|---|---|---|---|
| P | Narcs | 100 | 100 |
| P | Widgets | 250 | 150 |
| Q | Narcs | 200 | 100 |
| Q | Widgets | 300 | 150 |

The shipping cost per unit (Narcs or Widgets) from plant A to customers P and Q is $10 and $5, respectively, **and the shipping cost from plant B to customers P and Q is $5 and $15**, respectively, both for Narcs and Widgets. The selling price per unit for Narcs is $90. Widgets sell for $50. Nutworks has guaranteed to fill each order within the planning horizon. If a product has to be backordered, however, Nutworks will give a 10% discount per unit backordered.

Construct the symbolic LP(linear programming) model for the problem. Explicitly define the decision variables, objective function and constraints.

**2- Large Scale IP Problem:**

Undergraduate Flight Training (UFT) graduates in the US Air Force are required upon graduation to take advanced flight training and survival training courses enroute to their first operational assignment. Advanced flight training is offered only in formal schools usually by the Major air command, the principal aircraft user. UFT graduates must also take from one to four survival courses before being assigned to a crew, for example basic survival (Washington), water survival(Florida), air weapons delivery (Texas), etc. These courses are only offered at certain times, have enrolment limits and may have prerequisites. The identification of the schedules is further complicated by attendance requirements at combat Crew Training courses, various modes of transportation, the number of dead days in the pipeline, and the opportunity for the UFT graduates to take leave as desired, and so on.

To solve this UFT graduate scheduling problem, Air Force developed a computer program (called UFT Pipeline Scheduling Model) which generates from one to five feasible least cost schedules for each graduate. Using these schedules and course enrolment limits, the personnel manager in the Training Pipeline Management division manually assigns each graduate to one of his/her feasible schedules. Clearly, this is a difficult and time consuming task to do by hand; further, total cost of these manual assignments may be far from optimal.

The UFT problem may be modeled as a BIP(binary integer programming) model as well as a network model. Each graduate is assigned only to one schedule and each schedule has a pre-determined set of classes. Thus if a particular schedule is "selected", then every class in the schedule is automatically selected. The objective is to pick a schedule for each graduate that will minimize the cost of the assignments for the overall program, subject to the upper and lower attendance limits for each class.

You could use the following notation as well as define additional notation if necessary:
$c_{ij}$: cost of assigning graduate i to his/her $j^{th}$ schedule
$b_k$,$e_k$: lower and upper attendance limits for class k, respectively.
$N_i$ : set of schedule alternatives for graduate i
$S_k$: set of schedule alternatives containing class k

a) Formulate BIP model for the UFT problem. Explicitly state the decision variables, objective function and constraints.
b) If the UFT problem instance has 1200 graduates, 2500 classes and 9460 schedules, then what would be the number binary variables and constraints?

**3- Queuing Theory**
(You can use the tables provided at the back)

At the Forrester Manufacturing Company, one repair technician has been assigned the responsibility of maintaining three machines. For each machine, the probability distribution of the running time before a breakdown is exponential, with a mean of 9 hours. The repair time also has an exponential distribution, with a mean of 2 hours.

a) Which queuing model fits this queuing system?

b) Use this queuing model to find the probability distribution of the number of machines not running, and the mean of the distribution.

c) Use this mean to calculate the expected time between a machine breakdown and the completion of the repair of that machine.

d) What is the expected fraction of time that the repair technician will be busy?

e) As a crude approximation, assume that the calling population is infinite and that machine breakdowns occur randomly at a mean rate of 3 every 9 hours. Compare the result from part (b) with that obtained by making this approximation while using (i) the M/M/s model and (ii) the finite queue variation of the M/M/s model with K=3.

f) Repeat part (b) when a second repair technician is made available to repair a second machine whenever more than one of these three machines require repair.

## 4- Decision Analysis

A five-year-old pharmaceutical company faces a major research and development decision. It already has spent a year of preliminary research toward producing a protein that dissolves blood clots. Such a drug would be of tremendous value in the treatment of heart attacks, some 80 percent of which are caused by clots. The primary method the company has been pursuing relies on conventional, state-of-the-art biochemistry. Continuing this approach will require an estimated \$10 million additional investment and should lead to a commercially successful product, although the exact profit is highly uncertain; it is forecasted that a large success would lead to \$90 million in profits excluding cost of R&D with 70% probability or to a small success worth \$50 million in profits excluding R&D. Two of the company's most brilliant research scientists are aggressively advocating a second R&D approach. This new biogenetic method relies on gene splicing to create a version of the human body's own anticlotting agent and is considerably riskier than the biochemical alternative. It will require a \$20 million investment and has only a 20 percent chance of commercial success. However, if the company accomplishes the necessary breakthroughs, the anticlotting agent will represent the first blockbuster, genetically engineered drug. If successful, the method will entail minimal production costs and generate annual profits two to five times greater than a biochemically based drug would (approximate estimate is \$200 million in profits excluding R&D).

a) If the company were to choose between these two alternatives, which method should it choose for its R&D investment?

b) In the last shareholders' meeting, a proposition was made to develop both of the methods simultaneously. Would you recommend this over the choice in part (a).

c) Rather than simultaneously developing the biogenetic and biochemical alternatives, it is possible to develop them sequentially (i.e. first biochemical followed by biogenetic and vice-versa). Would you recommend this sequential development strategy over the strategies in parts (a) and (b).

**5- Sensitivity Analysis**

Consider the following problem

Minimize $W= 5y_1+4y_2$

Subject to

$4y_1+3y_2 \geq 4$
$2y_1+1y_2 \geq 3$
$y_1+2y_2 \geq 1$
$y_1+y_2 \geq 2$

and $y_1,y_2 \geq 0$

Because this primal problem has more functional constraints than variables, suppose that the simplex method has been applied directly to its dual problem. If we let $x_5$ and $x_6$ denote the slack variables for this dual problem, the resulting final simplex tableau is:

| | | Coefficient of: | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Basic Variable | Eq | Z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | RHS |
| Z | 0 | 1 | 3 | 0 | 2 | 0 | 1 | 1 | 9 |
| $x_2$ | 1 | 0 | 1 | 1 | -1 | 0 | 1 | -1 | 1 |
| $x_4$ | 2 | 0 | 2 | 0 | 3 | 1 | -1 | 2 | 3 |

For each of the following independent changes in the original primal model, you now are to conduct sensitivity analysis by directly investigating the effect on the dual problem and then inferring the complementary effect on the primal problem.

a) What are the shadow prices of the constraints of the primal problem?
b) What is the optimal basic solution of the primal problem?
c) What happens when we change the objective function to $W=9y_1+4y_2$
d) What happens when we change the third constraint's right hand side to 3? (i.e. $y_1+2y_2 \geq 3$)
e) Find the allowable range of values for the objective function coefficient of $y_2$?

### Template for M/M/s Finite Calling Population Model

| | Data | Data | Data | Data |
|---|---|---|---|---|
| $\lambda$ = | 0.3333 | 1 | 0.66666 | 0.166666 |
| $\mu$ = | 0.5 | 2 | 1 | 0.333333 |
| s = | 1 | 1 | 1 | 1 |
| N = | 3 | 3 | 3 | 3 |
| L = | 1.255813 | 1.421053 | 1.701483 | 1.42105 |
| $L_q$ = | 0.093023 | 0.631579 | 0.835814 | 0.631577 |
| W = | 2.16 | 0.9 | 1.965511 | 5.4 |
| $W_q$ = | 0.16 | 0.4 | 0.965511 | 2.399997 |
| $\rho$ = | 0.999999 | 1.5 | 1.99998 | 1.499995 |
| $\lambda$-bar = | 0.581395 | 1.578947 | 0.865669 | 0.263157 |

| n | Pn | Pn | Pn | Pn |
|---|---|---|---|---|
| 0 | 0.209303 | 0.210526 | 0.134331 | 0.210527 |
| 1 | 0.418605 | 0.315789 | 0.268659 | 0.31579 |
| 2 | 0.27907 | 0.315789 | 0.358208 | 0.315789 |
| 3 | 0.093023 | 0.157895 | 0.238803 | 0.157894 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |

### Template for M/M/s Finite Queue Model

| | Data | Data | Data | Data |
|---|---|---|---|---|
| $\lambda$ = | 0.3333 | 1 | 0.66666 | 0.166666 |
| $\mu$ = | 0.5 | 2 | 1 | 0.333333 |
| s = | 1 | 1 | 1 | 1 |
| K = | 3 | 3 | 3 | 3 |
| L = | 1.015275 | 0.733333 | 1.015374 | 0.733331 |
| Lq = | 0.430702 | 0.266667 | 0.430763 | 0.266665 |
| W = | 3.4736 | 0.7857 | 1.7368 | 4.7143 |
| Wq = | 1.473561 | 0.285714 | 0.736836 | 1.714283 |
| $\rho$ = | 0.6666 | 0.5 | 0.66666 | 0.499998 |

| n | Pn | Pn | Pn | Pn |
|---|---|---|---|---|
| 0 | 0.415427 | 0.533333 | 0.415389 | 0.533335 |
| 1 | 0.276924 | 0.266667 | 0.276923 | 0.266666 |
| 2 | 0.184597 | 0.133333 | 0.184614 | 0.133333 |
| 3 | 0.123052 | 0.066667 | 0.123074 | 0.066666 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 |

## Template for the M/M/s Queueing Model

$\lambda$ = (mean arrival rate)
$\mu$ = (mean service rate)
s = (# servers)
K = (max customers)
N = (size of population)

| | Data | Data | Data | Data | Data | Data | Data | Data |
|---|---|---|---|---|---|---|---|---|
| $\lambda$ = | 0.3333 | 1 | 0.66666 | 0.166666 | 0.3333 | 1 | 0.66666 | 0.166666 |
| $\mu$ = | 0.5 | 2 | 1 | 0.333333 | 0.5 | 2 | 1 | 0.333333 |
| s = | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| L = | 1.9994 | 1 | 1.99994 | 0.999994 | 0.749906 | 0.533333 | 0.749991 | 0.533332 |
| Lq = | 1.3328 | 0.5 | 1.33328 | 0.499996 | 0.083306 | 0.033333 | 0.083331 | 0.033333 |
| W = | 5.9988 | 1 | 2.99994 | 5.999988 | 2.249944 | 0.533333 | 1.124997 | 3.200002 |
| Wq = | 3.9988 | 0.5 | 1.99994 | 2.999985 | 0.249944 | 0.033333 | 0.124997 | 0.199999 |
| $\rho$ = | 0.6666 | 0.5 | 0.66666 | 0.499998 | 0.3333 | 0.25 | 0.33333 | 0.249999 |

| n | Pn | Pn | Pn | Pn | Pn | Pn | Pn | Pn |
|---|---|---|---|---|---|---|---|---|
| 0 | 0.3334 | 0.5 | 0.33334 | 0.500002 | 0.500038 | 0.6 | 0.500004 | 0.600001 |
| 1 | 0.222244 | 0.25 | 0.222224 | 0.25 | 0.333325 | 0.3 | 0.333332 | 0.3 |
| 2 | 0.148148 | 0.125 | 0.148148 | 0.125 | 0.111097 | 0.075 | 0.11111 | 0.075 |
| 3 | 0.098756 | 0.0625 | 0.098764 | 0.0625 | 0.037029 | 0.01875 | 0.037036 | 0.01875 |
| 4 | 0.06583 | 0.03125 | 0.065842 | 0.03125 | 0.012342 | 0.004688 | 0.012345 | 0.004687 |
| 5 | 0.043883 | 0.015625 | 0.043894 | 0.015625 | 0.004113 | 0.001172 | 0.004115 | 0.001172 |
| 6 | 0.029252 | 0.007813 | 0.029263 | 0.007812 | 0.001371 | 0.000293 | 0.001372 | 0.000293 |
| 7 | 0.019499 | 0.003906 | 0.019508 | 0.003906 | 0.000457 | 7.32E-05 | 0.000457 | 7.32E-05 |
| 8 | 0.012998 | 0.001953 | 0.013005 | 0.001953 | 0.000152 | 1.83E-05 | 0.000152 | 1.83E-05 |
| 9 | 0.008665 | 0.000977 | 0.00867 | 0.000977 | 5.08E-05 | 4.58E-06 | 5.08E-05 | 4.58E-06 |
| 10 | 0.005776 | 0.000488 | 0.00578 | 0.000488 | 1.69E-05 | 1.14E-06 | 1.69E-05 | 1.14E-06 |